Extra Long Baseline Neutrino Oscillations and CP Violation

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Abstract

The potential for studying CP violation in neutrino oscillations using conventional ν_μ and $\bar{\nu}_\mu$ beams is examined. For $\Delta m^2_{21} << \Delta m^2_{31}$ and fixed neutrino energy, E_ν, the CP violating asymmetry $A \equiv P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)/P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)$ in vacuum is shown to be measurable with roughly equal maximal statistical precision at distances $L_n \simeq (2n+1)\left(\frac{2\pi E_\nu}{\Delta m^2_{31}}\right), n = 0, 1, 2 \ldots$ (up to some n in the leading Δm^2_{21} approximation). For extra long baselines, $n \geq 1$, the falloff in detected oscillation events, $N = N_{\nu_e} + N_{\bar{\nu}_e}$, by $\sim 1/(2n+1)^2$ is compensated by a factor $\sim (2n+1)$ increase in the asymmetry such that the statistical figure of merit F.O.M. $\equiv (\delta A/A)^{-2} = A^2 N/1 - A^2$ is approximately independent of n. However, for the larger $n \geq 1$ asymmetries, some backgrounds as well as systematic uncertainties from flux normalization, detector acceptance etc. which scale with distance as $1/(2n+1)^2$ are shown to be relatively suppressed in $\delta A/A$ by $\sim 1/(2n+1)$. Also, low energy $E_\nu \simeq \mathcal{O}(1\text{GeV})$ extra long baseline experiments with $L_n \simeq 1200$–2900 km (corresponding to $\Delta m^2_{31} \simeq 3 \times 10^{-3}$ eV^2 and $n = 1$–3) are better matched to the remote locations of some proposed very large proton decay detectors which would be essential for neutrino CP violation studies. Effects of matter and realistic neutrino beam energy spread on extra long baseline CP violation experiments are briefly discussed.

Experimental evidence for neutrino masses, mixing and oscillations has become compelling. Atmospheric neutrino flux measurements by the Super-Kamiokande (SK) collaboration [1] indicate large mixing $\nu_\mu \to \nu_\tau$ oscillations governed by a neutrino mass squared difference

$$\Delta m^2_{32} \equiv m^2_3 - m^2_2 \simeq (1.6 - 4) \times 10^{-3}\text{eV}^2 \tag{1}$$

(The sign of Δm^2_{32} is not yet determined, but here we assume $m_3 > m_2$.) Also, recent results [2] from SNO on the solar ν_e flux when combined with earlier SK and other [3] solar neutrino data strongly point to the so-called Large Mixing Angle (LMA) scenario with [4]

$$\Delta m^2_{21} \equiv m_2^2 - m_1^2 \simeq (2-10)\times 10^{-5}\mathrm{eV}^2 \tag{2}$$

In terms of the 3×3 mixing matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_1c_3 & s_1c_3 & s_3e^{-i\delta} \\ -s_1c_2 - c_1s_2s_3e^{i\delta} & c_1c_2 - s_1s_2s_3e^{i\delta} & s_2c_3 \\ s_1s_2 - c_1c_2s_3e^{i\delta} & -c_1s_2 - s_1c_2s_3e^{i\delta} & c_2c_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{3}$$

$$c_i \equiv \cos\theta_i \quad , \quad s_i \equiv \sin\theta_i \quad , \quad i = 1,2,3$$

those combined results imply relatively large mixing [5]

$$\begin{aligned} \sin\theta_1 &\simeq 0.4 \\ \sin\theta_2 &\simeq 0.707 \end{aligned} \tag{4}$$

On the other hand, failure to observe oscillations in $\bar{\nu}_e \to \bar{\nu}_x$ reactor searches gives the bound [6] (for $\Delta m^2_{31} \simeq 3\times 10^{-3}$ eV^2)

$$\sin^2 2\theta_3 < 0.12 \qquad \text{or} \qquad s_3 < 0.18 \qquad (90\%\ \text{CL}) \tag{5}$$

suggesting small first-third generation mixing. In the case of the phase, δ, nothing is currently known. Determination of δ and θ_3 will tell us the amount of CP violation inherent to lepton charged current mixing via the Jarlskog CP violation invariant [7]

$$J_{CP} \equiv s_1s_2s_3c_1c_2c_3^2\sin\delta \tag{6}$$

Future experimental efforts will aim to measure the above parameters with high precision and search for potential deviations from expectations due to "new physics" such as sterile neutrino mixing. Towards those ends, the KamLAND reactor experiment [8] should pin down the values of both Δm^2_{21} and $\sin\theta_1$, if the LMA solar neutrino solution is indeed correct. Similarly,

$K2K$ [9] and MINOS [10] should better determine Δm_{32}^2 and $\sin\theta_2$ via ν_μ disappearance. Further down the road, a recent proposal [5] for a JHF-Kamioka $\nu_\mu \to \nu_e$ oscillation experiment using a conventional horn generated pion decay neutrino beam of $\sim$ 0.7 GeV from their future 0.77 megawatt proton synchrotron and the large (22kton) SK detector located 295 km away would search for $\sin^2 2\theta_3$ down to 0.006 (a factor of 20 improvement over the current bound) and measure Δm_{32}^2 to $\pm 10^{-4}$ eV^2 and $\sin^2 2\theta_2$ to about $\pm 1\%$.

The reach of the JHF-Kamioka proposal [5] illustrates the significant potential of intense low energy conventional neutrino beams for studying oscillations when a very large detector such as SK is available. The next generation of proton decay detectors will have fiducial volumes 20–40 times larger than SK and will probably provide our first opportunity to probe CP violation in neutrino oscillations and thereby determine the phase δ. Indeed, the JHF-Kamioka neutrino project [5] envisions a phase II upgrade of its accelerator complex to 4 Megawatts, which when combined with a possible 1000 kton Hyper-Kamiokande detector, could increase their yearly statistics by a factor of 200. Studies [5] indicate that with such a facility, in 2 years of ν_μ and 6 years of $\bar{\nu}_\mu$ running, they would statistically probe CP violation well below $\delta \simeq \pi/9$, assuming systematic uncertainties in the comparison of $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ and backgrounds can be controlled.

In this paper, we examine potential advantages of studying CP violation with conventional ν_μ and $\bar{\nu}_\mu$ beams at extra long baseline distances

$$L_n \simeq (2n+1)L_0 \quad , \quad n = 1, 2, 3 \ldots \tag{7a}$$

which are larger by odd multiples 3,5,7 etc. than the generally considered first oscillation maximum

$$L_0 = \left(\frac{2\pi E_\nu}{\Delta m_{31}^2}\right) \simeq 413(E_\nu/\mathrm{GeV}) \left(\frac{3 \times 10^{-3}\mathrm{eV}^2}{\Delta m_{31}^2}\right) km \tag{7b}$$

As we will show, to leading order in $\Delta m_{21}^2/\Delta m_{31}^2 << 1$, the statistical figure of merit (F.O.M.)

$$F.O.M. \equiv (\delta A/A)^{-2} \tag{8}$$

which describes ones ability to measure A, J_{CP} and $\sin\delta$ is roughly independent of n (up to some n where the leading Δm_{21}^2 approximation starts to fail). That finding implies that CP violation experiments at the first few $L_n, n = 0, 1, 2 \ldots$, are (roughly) statistically equivalent to first approximation (until one runs out of events). However, the extra long baselines with $n \geq 1$ have a potentially significant advantage when it comes to backgrounds and systematic uncertainties from neutrino flux normalization, detector acceptance etc. which scale down by $1/(2n+1)^2$ with the larger distances. Such effects are relatively suppressed in $\delta A/A$ by $\sim 1/2n+1$. Furthermore, the longer distances may be better matched to the remote locations of very large proton decay detectors [11] which are unlikely to be located $\lesssim 413(E_\nu/\mathrm{GeV})\left(\frac{3\times10^{-3}\mathrm{eV}^2}{\Delta m^2_{31}}\right)$ km from existing intense sources of $E_\nu \sim \mathcal{O}(1 \text{ GeV})$ neutrino beams such as the AGS at Brookhaven or the Fermilab Booster. For example Brookhaven-Homestake = 2540km Fermilab-Homestake = 1290km while Brookhaven-Carlsbad = 2920km and Fermilab-Carlsbad = 1770km [12]. (An exceptional case is the JHF-Hyper-Kamioka proposal [5] which is fixed at $\sim$ 295 km and thus optimized for and constrained to $E_\nu \simeq 0.7\left(\frac{3\times10^{-3}\mathrm{eV}^2}{\Delta m^2_{31}}\right)$ GeV CP violation studies.)

We begin by giving the exact $\nu_\mu \to \nu_e$ oscillation probability in vacuum at a distance L for neutrino energy E_ν (matter effects are subsequently considered). In terms of the mixing angles in (3), one finds (for $\bar\nu_\mu \to \bar\nu_e$ and $\nu_e \to \nu_\mu$, $J_{CP} \to -J_{CP}$)

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= 4(s_2^2 s_3^2 c_3^2 + J_{CP}\sin\Delta_{21})\sin^2\frac{\Delta_{31}}{2} \\ &\quad +2(s_1 s_2 s_3 c_1 c_2 c_3^2\cos\delta - s_1^2 s_2^2 s_3^2 c_3^2)\sin\Delta_{31}\sin\Delta_{21} \\ &\quad +4(s_1^2 c_1^2 c_2^2 c_3^2 + s_1^4 s_2^2 s_3^2 c_3^2 - 2s_1^3 s_2 s_3 c_1 c_2 c_3^2\cos\delta - J_{CP}\sin\Delta_{31})\sin^2\frac{\Delta_{21}}{2} \\ &\quad +8(s_1 s_2 s_3 c_1 c_2 c_3^2\cos\delta - s_1^2 s_2^2 s_3^2 c_3^2)\sin^2\frac{\Delta_{31}}{2}\sin^2\frac{\Delta_{21}}{2} \end{aligned} \tag{9}$$

where J_{CP} is given in (6) and

$$\begin{aligned} \Delta_{31} &\equiv \Delta m^2_{31} L/2E_\nu \\ \Delta_{21} &\equiv \Delta m^2_{21} L/2E_\nu \end{aligned} \tag{10}$$

For studies of CP violation, one considers the asymmetry

$$A \equiv \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)} \tag{11}$$

To leading order in Δ_{21} (assumed to be small), one finds

$$P(\nu_\mu \to \nu_e) \simeq 4 s_2^2 s_3^2 c_3^2 \sin^2 \frac{\Delta_{31}}{2} + \mathcal{O}(\Delta_{21}) \tag{12a}$$

$$A \simeq \frac{J_{CP} \sin \Delta_{21}}{s_2^2 s_3^2 c_3^2} \simeq \frac{2 s_1 c_1 c_2 \sin \delta}{s_2 s_3} \left(\frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right) \frac{\Delta m_{31}^2 L}{4 E_\nu} + \mathcal{O}(\Delta_{21}^2) \tag{12b}$$

The asymmetry grows linearly with distance, but for fixed detector size and neutrino energy, the flux of neutrinos decreases as $\sim 1/L^2$. The experimental determination of A will entail measuring N_{ν_e} events from $\nu_\mu \to \nu_e$ and $N_{\bar{\nu}_e}$ from $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations such that

$$A^{\rm exp} = \frac{N_{\nu_e} - r N_{\bar{\nu}_e}}{N_{\nu_e} + r N_{\bar{\nu}_e}} \tag{13}$$

where r is the ratio of (ν_μ flux)$\times$(ν_e cross-section)$\times$($\nu_e N \to e^- N'$ acceptance)$\times$ (running time) divided by the same quantity for antineutrinos. The running times would normally (for small asymmetries) be arranged to make $r = 1$ up to systematic uncertainties. (For large asymmetries, a different running strategy might be employed in which run times are further relatively weighted by $\frac{1+A}{1-A}$.)

For our statistical discussion, we set $r = 1$ and find the statistical figure of merit (F.O.M.) regarding how well the asymmetry and therefore $\sin \delta$ are measured is given by

$$\begin{aligned} F.O.M. &= (\delta A/A)^{-2} = A^2 N/1 - A^2 \\ N &= N_{\nu_e} + N_{\bar{\nu}_e} \end{aligned} \tag{14}$$

Note that it is linear in N and quadratic in A. From (12), one finds to leading order in Δ_{21} that the statistical F.O.M. is optimized at the oscillation maxima

$$\frac{\Delta_{31}}{2} = \frac{\Delta m_{31}^2 L_n}{4 E_\nu} = (2n+1)\pi/2 \tag{15}$$

At distances L_n, the growth in A by a factor of $(2n+1)$ compensates for the reduction in N by $1/(2n+1)^2$ due to flux reduction, leaving the F.O.M. roughly independent of n (at least until the small Δ_{21} approximation breaks down). Where our approximation starts to break down depends on s_3 and $\Delta m^2_{21}/\Delta m^2_{31}$ as our examples will subsequently illustrate. Note also, that in our approximation the F.O.M. for $\delta A/A$ is insensitive to the value of s_3, at least until s_3 becomes so small that the approximation in (12a) breaks down.

To illustrate the above finding and its domain of validity, we consider measurements carried out at different L_n for $E_\nu = 1$ GeV beams employing the following realistic parameters

$$\begin{array}{cc} s_1 = 0.4 & c_1 = 0.916 \\ s_2 = 0.707 & c_2 = 0.707 \\ s_3 = 0.10 & c_3 = 0.995 \\ \sin\delta = 0.707 & \cos\delta = 0.707 \end{array}$$

$$\Delta m^2_{31} = 3\times 10^{-3}\mathrm{eV}^2 \qquad \Delta m^2_{21} = 5\times 10^{-5}\mathrm{eV}^2$$

in the full expression of eq. (9). For definiteness, we assume running times (2 years of ν_μ and 6 years of $\bar{\nu}_\mu$), fluxes and detector assumptions given in phase II of the JHF-Hyper-Kamiokande proposal [5] which envisions a 1000 kton water cerenkov detector and 4 megawatt proton synchrotron. We assume cuts on the events that reduce the oscillation signals by about a factor of 2. In that way, our statistics roughly represent simple scaling of the JHF-Hyper-Kamiokande proposal. In an actual experiment unconstrained by the 295 km requirement, one would probably employ somewhat higher (average) energy neutrino beams, looser cuts and perhaps a wide band beam at larger distances. However, those potential gains in statistics might be somewhat offset by oscillation losses at larger distances (see subsequent discussion); so, our table numbers should be fairly realistic but are not to be taken too literally.

Table 1 nicely illustrates the main point of this paper. The statistical F.O.M. is rather constant for $n = 0$–3. Beyond those distances, the Δm^2_{21} governed oscillation starts to become significant and eventually begins to reduce the asymmetry (although it remains quite appreciable). Of course, at the larger distances, matter effects and beam characteristics become more important and should be carefully folded into any distance-beam energy optimization.

n	$L_n(km)$	$N_{\nu_e} + N_{\bar{\nu}_e}$	A	F.O.M.
0	413	16000	0.134	293
1	1239	1952	0.366	302
2	2065	824	0.516	299
3	2891	512	0.586	268
4	3717	384	0.601	217
5	4543	315	0.585	164

Table 1: Number of total oscillation events (after cuts) $N_{\nu_e} + N_{\bar{\nu}_e}$, the CP violating asymmetry A, and F.O.M. at different distances for the mixing parameters given above. $E_\nu \simeq 1$ GeV, $r = 1$ and the experimental conditions given in the text are assumed. A F.O.M $= 293$ corresponds to about a 17 sigma determination of the asymmetry A.

To demonstrate the robustness of our result, we give in table 2 changes brought about by reducing s_3 from 0.1 to 0.05. Note, that the oscillation statistics are reduced, but the F.O.M. is rather unaffected (at least for $n = 0$–2). Here the dilution due to Δ_{21} oscillations starts to set in earlier.

n	$L_n(km)$	$N_{\nu_e} + N_{\bar{\nu}_e}$	A	F.O.M.
0	413	4168	0.257	295
1	1239	616	0.580	312
2	2065	328	0.650	240
3	2891	248	0.600	140
4	3717	216	0.536	87

Table 2: Parameters the same as table 1 except $s_3 = 0.05$ instead of 0.10.

The measurement of CP violation as illustrated in tables 1 and 2 is statistically significant, ~ 17 sigma for the first few n. One could half the running time, descope the large water cerenkov detector to a more manageable 500kton, go from a 4 to a 1 Megawatt proton synchrotron and still have a robust 4 sigma signal. However, a reduced asymmetry due to a much smaller $\sin\delta \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$ than the 0.012 value assumed here would require the higher statistics. One should have a better idea regarding experimental demands as Δm_{21}^2, Δm_{31}^2 and the θ_i are more precisely determined.

Where the extra long baselines have a potential significant advantage is in the area of systematic errors from flux normalization, detector acceptance and those backgrounds that scale with distance as $1/(2n+1)^2$. To illustrate that point, we consider the experimental asymmetry in (13) where $r \simeq 1$ contains uncertainties from flux, cross-section, acceptance and other normalization effects. The fractional systematic error $\delta r/r$ translates into the following asymmetry error

$$\delta A/A \simeq \frac{1}{2}\,\frac{1-A^2}{A}\,\frac{\delta r}{r} \tag{16}$$

For larger asymmetries, the effect of $\delta r/r$ is significantly reduced. For asymmetries exhibiting the $(2n+1)$ growth, such systematics are reduced by $\sim 1/2n+1$. This advantage can be significant, particularly if the CP violating asymmetry turns out to be much smaller than in our previous illustrative examples.

Other backgrounds, such as $\overset{(-)}{\nu}_e$ beam contamination and misidentified neutral current residual events remaining after cuts, that scale down by $1/L^2$ will similarly be reduced in relative importance by $\sim 1/A$ for larger asymmetries. Non-distance dependent effects such as cosmic ray backgrounds would of course be worse for the reduced statistics of extra long baseline experiments. Fortunately, they are generally very small in deep underground experiments because of directionality and the precise timing constraints imposed by beam pulsing. Nevertheless, such effects are potentially more dangerous and must be carefully studied.

For realistic experimental considerations, the full oscillation formula in (9) supplemented with matter effects [13] must be employed. In addition, neutrino beam energy spread, detector acceptance, backgrounds etc. must be considered. Here, we make some brief qualitative observations concerning such effects. They will be further examined in a subsequent more detailed study.

At extra long distances, L_n, $n = 1$–3, the smaller frequency Δ_{21} oscillations and matter effects will effectively shift somewhat the oscillation peaks. More important, matter effects will induce a fairly significant CP non-violating (positive) asymmetry which must be well controlled and understood in any serious study. For the distances considered here $\sim$ 1200–2900 km and neutrino energies $E_\nu \sim \mathcal{O}(1 \text{ GeV})$, such calculations can be carried out in a straightforward way that exactly includes Δm^2_{31}, Δm^2_{21} and matter density effects [14]. Such a study has been nicely illustrated in ref. 15. There, one finds for a distance of 2900 km that matter effects are most important for the $n = 0$ peak at $E_{\nu_\mu} \simeq 6$ GeV. At lower energies, ν_μ oscillation peaks are also somewhat shifted but less amplified due to Δm^2_{21} and matter effects. Those shifts may influence the experimental beam energy setting for a given baseline distance, but they should not significantly change our discussion about the statistical significance of extra long oscillation studies near the first few L_n with $n = 1, 2, 3$ as long as E_ν is not too large. The CP violating asymmetry increase at the longer distances should continue to compensate for the flux falloff in the F.O.M. at least for the first few $n \geq 1$. If the matter asymmetry and CP violating asymmetry have the same sign (both positive), we can have potentially enormous combined asymmetries at long distances with the $1-A^2$ in the F.O.M. denominator playing a significant role. Alternatively, for opposite signs, matter and CP violation asymmetries will partially cancel. Of course, such a cancellation is also possible and potentially more problematic for $n = 0$. (In the JHF-Kamioka discussion of ref. 5, a partial cancellation of matter and CP violating asymmetries which dilutes the signal is apparent for the phase chosen as an illustration $\delta = -45°$.) Generally, one expects better control of matter induced asymmetries for the larger CP violating asymmetries of extra long, $n \geq 1$, baselines.

Also of importance for extra long beamline experiments is the neutrino energy spectrum [16]. For conventional pion decay generated neutrino beams from 16 $\sim$ 28 GeV proton drivers, one can either employ a narrow band (energy spread) beam centered at some energy $E_\nu \sim 1$ GeV by placing the detector off-beam axis by some small angle or opt for a more intense wide band beam with considerable support in the $E_\nu \simeq 0.5$–2 GeV range [5, 16]. Because the oscillation peaks at $n \geq 1$ have a narrower energy width, it will be more challenging to tailor a narrow band beam to the detector distance and oscillation parameters as n grows. Instead, the wide band neutrino beam may have advantages. In addition to the higher flux of neutrinos, the energy spread would simultaneously cover several oscillation maxima. For example, if the $E_\nu \simeq 2$ GeV component corresponds to the $n = 1$ peak (for $L_1 \simeq 2500$km), there will also be potentially observable peaks at E_ν approximately 1.2 GeV ($n = 2$), 0.86 GeV ($n = 3$) and 0.67 GeV ($n = 4$). Of course, there will also be oscillation minima between the peaks that would reduce statistics (roughly wiping out some flux gains of a wide band beam). Nevertheless, an experiment that covers and can partially unravel the $n = 1$–4 oscillation peaks has the potential (if statistics suffice) to not only probe CP violation, but to determine the sign of Δm^2_{31} and study the changing matter effects in detail. Such potential capabilities provide strong motivation for extra long baseline experiments. Those features will be explored in detail in a subsequent study.

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